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Elastic Constants for Superplastically Formed/ Diffusion-Bonded Sandwich Structures

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Abstract

FORMULAS for evaluating the effective elastic constants for a superplastically formed/diffusion-bonded (SPF/DB) unidirectionally corrugated core sandwich structure like that shown in Fig. 1 are presented. This structure is formed by diffusion bonding three superplastic alloy sheets (two face sheets and one core sheet) in the selected areas and then superplastically expanding the multiple sheet pack inside a die cavity by using gas pressure.[1] Thus, this structure is slightly different from the conventional corrugated-core sandwich structure because the corrugation leg does not have uniform thickness. Because of superplastic expansion, the diagonal segment of the corrugation leg is always thinner than the flat segment (crest or trough) of the corrugation leg, which has a thickness that is nearly the pre-expansion thickness. Thus, the results given by Ref. 2 for a corrugated-core sandwich plate cannot be used in the present structure without considerable modification.

In the analysis, only a symmetric corrugation (i.e., a corrugation symmetric with respect to the middle surface of the sandwich plate) is considered, and the corrugation leg is assumed to be made up of two corrugation flat segments of thickness t_f (the pre-expansion thickness of the core sheet), two circular arc segments, and a straight diagonal segment. The circular and straight diagonal segments have the same thickness t_c.

Contents

The elastic constants evaluated for the SPF/DB unidirectionally corrugated sandwich core are the effective (or averaged) elastic constants for an equivalent homogeneous sandwich core representing the actual sandwich core. This idealization can be adequate when the sandwich plate width (normal to the corrugation axis) is many times greater than the corrugation pitch. The analysis assumes that the deformation is infinitesimal without local buckling of the sandwich structure.

Elastic Constants E_x, G_{xy}, and G_{zx}

The modulus of elasticity E_x and the shear moduli G_{xy} and G_{zx} may be obtained by modifying the results given in Ref. 2 through the introduction of the rule of mixture to account for the nonuniform thickness of the corrugation leg. These elastic constants can be expressed in the following forms:

$$E_x = E_c \frac{1}{ph_c}[ft_f + (\ell - f)t_c] = E_c \frac{t_f}{h_c} \tag{1}$$

$$G_{xy} = G_c \frac{p}{\ell^2 h_c}[ft_f + (\ell - f)t_c] = G_c \frac{p^2 t_f}{\ell^2 h_c} \tag{2}$$

$$G_{zx} = G_c \frac{h_c}{\ell^2 p}[ft_f + (\ell - f)t_c] = G_c \frac{h_c t_f}{\ell^2} \tag{3}$$

where the last terms in Eqs. (1-3) are for constant volume superplastic deformations (neglecting small elastic deformations) for which the condition $ft_f + (\ell - f)t_c = pt_f$ holds, and E_c is the modulus of elasticity of the corrugated core material, G_c the shear modulus of the corrugated core material, p is one-half the corrugation pitch (or half wavelength of corrugation), h_c the corrugated core thickness (vertical distance between the upper and lower corrugation flat segment centerlines), ℓ the length of one corrugation leg, and f the length of corrugation flat segment (crest or trough).

For constant volume superplastic deformations, the superplastic incompressibility condition used in Eqs. (1-3) may be used to relate the two corrugation leg thicknesses t_f and t_c through the following alternative forms:

$$\frac{t_c}{t_f} = \frac{p-f}{\ell - f} = \frac{b}{d+R\theta} \tag{4}$$

or

$$\frac{t_c}{t_f} = \frac{\cos\theta + 2(R/h_c)(1-\cos\theta)}{1+2(R/h_c)[\theta\sin\theta - (1-\cos\theta)]} \tag{5}$$

where b is one-half of the horizontal projected length of the corrugation leg, d is one-half of the length of the straight diagonal segment of the corrugation leg, R is the radius of the circular arc segment of the corrugation leg, and θ is the corrugation angle (angle between the centerline of the straight diagonal segment and that of the corrugation flat segment).

Elastic Constants E_y, E_z, G_{yz}, ν_{yz}, ν_{zx}, and D_x

Using elementary theories of strength of materials, the moduli of elasticity E_y and E_z, the shear modulus G_{yz}, the

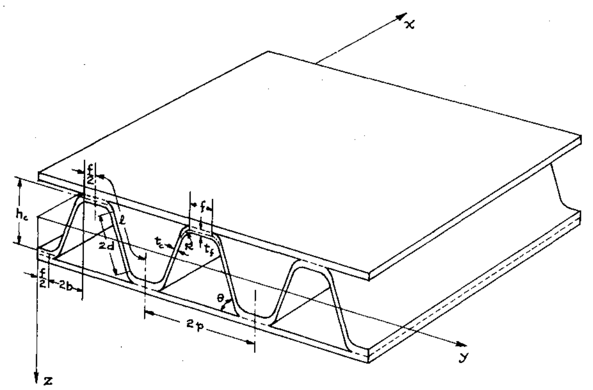

Fig. 1 Geometry of superplastically formed/diffusion-bonded sandwich plate with unidirectionally corrugated core.

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Poisson ratios ν_{yz} and ν_{zy}, and the bending stiffness D_x (per unit width of beam cut from the SPF/DB corrugated sandwich core in the x direction) may be expressed in the following forms (see Ref. 3 for detailed derivations):

$$E_y = (E_c/D_y^H)\,(I_c p/h_c^4) \tag{6}$$

$$E_z = (E_c/D_z^F)\,(I_c/p h_c^2) \tag{7}$$

$$G_{yz} = E_c I_c \Big/ \left[h_c^3 \left(\frac{h_c}{p} D_z^F - 2D_z^H + \frac{p}{h_c} D_y^H \right) \right] \tag{8}$$

$$\nu_{yz} = (h_c/p)\,(D_y^F/D_z^F) \tag{9}$$

$$\nu_{zy} = (p/h_c)\,(D_z^H/D_y^H) \tag{10}$$

$$D_x = E_c \bar{I}_c \tag{11}$$

where

$$D_y^H \equiv \frac{2}{3}\left(\frac{d}{h_c}\right)^3 \sin^2\theta + \frac{1}{2}\left(\frac{R}{h_c}\theta + \frac{1}{2}\frac{f}{h_c}\frac{I_c}{I_f}\right) - \left(\frac{R}{h_c}\right)^2 \left[\left(2 - 3\frac{R}{h_c}\right)(\theta - \sin\theta) + \frac{R}{h_c}\sin\theta\,(1-\cos\theta)\right] + \frac{I_c}{h_c^2 t_c}\left[\frac{f}{h_c}\frac{t_c}{t_f} + 2\frac{d}{h_c}\cos^2\theta + \frac{R}{h_c}(\theta + \sin\theta\cos\theta)\right] \tag{12}$$

$$D_z^H = \frac{2}{3}\left(\frac{d}{h_c}\right)^3 \sin\theta\cos\theta + \frac{1}{2}\frac{I_c}{I_f}\left[\frac{1}{4}\left(\frac{p}{h_c}\right)^2 - \left(\frac{b}{h_c}\right)^2\right] + \frac{R}{h_c}\left\{\frac{b}{h_c}\theta - 2\frac{Rb}{h_c^2}(\theta - \sin\theta) - \frac{R}{h_c}(1-\cos\theta)\left[1 - \frac{R}{h_c}(1-\cos\theta)\right]\right\} - \frac{I_c}{h_c^2 t_c}\left(2\frac{d}{h_c}\sin\theta\cos\theta + \frac{R}{h_c}\sin^2\theta\right) \tag{13}$$

$$D_y^F = D_z^H \tag{14}$$

$$D_z^F = \frac{2}{3}\left(\frac{d}{h_c}\right)^3 \cos^2\theta + \frac{2}{3}\frac{I_c}{I_f}\left[\frac{1}{8}\left(\frac{p}{h_c}\right)^3 - \left(\frac{b}{h_c}\right)^3\right] + \frac{R}{h_c}\left[2\left(\frac{b}{h_c}\right)^2\theta - 4\frac{Rb}{h_c^2}(1-\cos\theta) + \left(\frac{R}{h_c}\right)^2(\theta - \sin\theta\cos\theta)\right] + \frac{I_c}{h_c^2 t_c}\left[2\frac{d}{h_c}\sin^2\theta + \frac{R}{h_c}(\theta - \sin\theta\cos\theta)\right] \tag{15}$$

$$I_c = (1/12)\,t_c^3 \tag{16}$$

which is the moment of inertia, per unit width, of the cross section of the corrugation leg segment of thickness t_c, taken with respect to the horizontal neutral axis (parallel to the x axis) of the cross section, and finally

$$\bar{I}_c = \frac{h_c^3 t_c}{p}\left\{\frac{1}{4}\frac{f}{h_c}\frac{t_f}{t_c}\left(1 + \frac{1}{3}\frac{t_f^2}{h_c^2}\right) + \frac{2}{3}\frac{d^3}{h_c^3}\left(\sin^2\theta + \frac{1}{4}\frac{t_c^2}{d^2}\cos^2\theta\right) + \frac{R}{h_c}\left[\frac{\theta}{2} - 2\frac{R^2}{h_c^2}\sin\theta - \frac{R}{h_c}\left(2 - 3\frac{R}{h_c}\right)(\theta - \sin\theta)\right]\right\} \tag{17}$$

which is the moment of inertia, per unit width, of the corrugation cross section parallel to the y,z plane, taken about the horizontal centroidal axis of the corrugation cross section.

The bending stiffness in the y direction of the SPF/DB corrugated core is very small and is of the order of $E_c t_i^3/12$ (where $t_i = t_f$ or t_c).

Effect of Face Sheets on E_z

If the corrugated core is bounded to the face sheets of thickness t, the stiffness in the z direction (i.e., E_z) is greatly enhanced. If $\bar{E}_z$ denotes the modulus of elasticity in the z direction when the deformation of the corrugated core is constrained by the face sheets, then $\bar{E}_z$ can be expressed as follows (see Ref. 3):

$$\bar{E} = E_z \Big/ \left\{ 1 - \frac{\nu_{yz}\nu_{zy}}{1 + (pI_c)/(2th_c^3 D_y^H)} \right\} \tag{18}$$

For a given corrugation angle and corrugation leg thickness, the thickness extensional stiffness, the bending stiffness, and the shear stiffness in the plane normal to the corrugation axis are quite sensitive to the change in the dimension of the corrugation crest (or trough; see the graphic presentations of the elastic constants in Ref. 3).

By assuming uniform thickness for the corrugation leg, the formulas developed could be applied directly to the evaluation of effective elastic constants for the honeycomb sandwich core that is made up of corrugation strips joined together at their crests and troughs (see Ref. 3).

References

[1] Pulley, J., "Evaluation of Low-Cost Titanium Structure for Advanced Aircraft," NASA CR-145111, 1976.

[2] Libove, C. and Hubka, R. E., "Elastic Constants for Corrugated-Core Sandwich Plates," NACA TN 2289, 1951.

[3] Ko, W. L., "Elastic Constants for Superplastically Formed/Diffusion-Bonded Sandwich Structures," AIAA Paper 79-0756, April 1979.